

Resonant Relaxation near the Massive Black Hole in the Galactic Center

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Abstract. The coherent torques between stars on orbits near massive black holes (MBHs) lead to resonant angular momentum relaxation. Due to the fact that orbits are Keplerian to good approximation, the torques efficiently change the magnitude of the angular momenta and rotate the orbital inclinations. As a result the stars are rapidly randomized. The galactic MBH is a good system for the observational study of resonant relaxation. The age of the young B-stars at a distance of ~ 0.01 pc from the MBH is comparable to the resonant relaxation time, implying that resonant relaxation may have played an important role in their dynamical structure. In contrast, the O-stars in the stellar disks at ~ 0.1 pc are younger than the resonant relaxation time, as required by their dynamical coherence. Resonant relaxation dynamics dominates the event rate of gravitational wave (GW) emission from inspiraling stars into MBHs of masses comparable to the Galactic MBH. Resonant relaxation leads to rates $\lesssim 10$ times higher than those predicted by 2-body relaxation, which would improve the prospects of detecting these events by future GW detectors, such as *LISA*.

1. Introduction

In “collisional” stellar dynamics the potential in which a star moves is considered to be smooth to first order, and the fact that the potential is in fact made out of discrete stars is treated as a perturbation (e.g. Chandrasekhar [11]; Binney & Tremaine [10]). Stars have orbital parameters such as angular momentum and energy that are conserved in the smooth potential, and these parameters indeed remain constant for many dynamical times t_d . Only the weak encounters allow for changes in these quantities. For example, two stars which interact with each other, can exchange energy and angular momentum, so that after their encounters their orbits are described by slightly different quantities. In this way not only the orbits of individual stars are modified, but the distribution function (DF) of the entire system can change, and evolve towards a steady state. The time-scale over which a system evolves is the *relaxation time* t_r . In most systems $t_d \ll t_r$, and relaxation can indeed be treated as a second order effect.

Analyses of the evolution of the DF near a MBH have almost exclusively relied on the assumption that the mechanism through which stars exchange angular momentum and energy is dominated by *uncorrelated two-body interactions*¹. Any encounter is assumed to be unrelated to previous and future encounters, and changes in energy and angular momenta are considered to be drawn from a specified random distribution. Relaxation can therefore in a meaningful way

¹ *N*-body simulations form an exception; for *N*-body simulations near MBHs see e.g. Baumgardt, Makino & Ebisuzaki [7], [8]; Preto, Merritt & Spurzem [44]; Merritt & Szell [37]

be considered to be a random walk process. In the context of stellar dynamics near MBHs, the assumption of uncorrelated encounters is made in Fokker-Planck models (e.g. Bahcall & Wolf [5]; Bahcall & Wolf [6]; Cohn & Kulsrud [12]; Murphy, Cohn & Durisen [39]), where the microscopic interactions are expressed by the diffusion coefficients, and in Monte Carlo simulations (e.g. Shapiro & Marchant [49]; Marchant & Shapiro [35], [36]; Freitag & Benz [15], [16]). Stars around MBHs are described as moving in the smooth average potential of the MBH and the stars, and the scattering by the fluctuating part of the potential is modeled as a hyperbolic Keplerian interaction between a passing star and a test star.

The (non-resonant) relaxation time T_{NR} can be defined as the time it takes for the energy \mathcal{E} of a typical star to change by order unity. This is also the time it takes for its specific angular momentum J to change by an amount of order $J_c(\mathcal{E})$, the maximal angular momentum for that energy. On Keplerian orbits $J_c = \sqrt{GM_\bullet a}$, where a is the semi-major axis. The “non-resonant” relaxation time T_{NR} of stars of mass M_\star can be written in the Keplerian regime as

$$T_{\text{NR}} = A_\Lambda \left(\frac{M_\bullet}{M_\star} \right)^2 \frac{P(a)}{N(< a)} \quad (M_\bullet \gg M_\star), \quad (1)$$

where $P = 2\pi\sqrt{a^3/(GM_\bullet)}$ is the orbital period and A_Λ is a dimensionless constant which includes the Coulomb logarithm. For some stellar systems (like galaxies), T_{NR} is much longer than the age of the system, implying that the system cannot evolve significantly towards steady state by two-body interactions. For other systems, like very dense stellar clusters, the relaxation time can be as small as a few Myr, and such systems may even evaporate within a Hubble time. In our GC, the relaxation time is somewhat smaller than the age of the system, $T_{\text{NR}} \sim \text{few Gyr}$ (e.g. Alexander [1], [4]), implying that the system has evolved considerably, and two-body relaxation effects such as mass-segregation have occurred (Bahcall & Wolf [6]; Freitag, Amaro-Seoane & Kalogera [19]; Hopman & Alexander [28]). At the same time, the relaxation time is much longer than some other relevant times in the GC, in particular the age of the youngest stars.

The assumption of uncorrelated two-body interactions is well-justified in many systems, such as globular clusters, where stellar orbits are not closed. However, the special symmetry of a Keplerian potential leads to closed, elliptical orbits. The fact that the orbits are closed can be exploited in numerical treatment, and also leads to unique dynamical features (see also the contribution of Touma in this volume). Since $t_r \gg t_d$, the orbits remain closed for many dynamical times, the system may be thought of as a set of “wires” with the mass of the star smeared out over the orbits. In this picture, it is the wires that interact and cause the evolution of the system, rather than point particles interacting at given locations. The idea is reminiscent of the Kozai mechanism in triple stars (Kozai [30]).

Rauch & Tremaine [45] first used this approach in the context of many body stellar dynamics near MBHs, and coined the term *resonant relaxation* (RR), after the 1 : 1 resonance between the radial and azimuthal frequencies in a Keplerian potential. The wire approximation is only relevant for times $\ll t_\omega$, where t_ω is the time for the orbit to precess. Precession may be caused by the fact that the potential is not entirely determined by a point mass, and there is still some extended component to the potential due to the stellar mass; this is especially the case far away ($\gtrsim 0.1$ pc) from the MBH. Closer to the MBH ($\lesssim 0.01$ pc), precession may be dominated by effects of General Relativity².

1.1. Scalar resonant relaxation

Scalar relaxation results in changes in *both* the direction and the magnitude of the angular momenta. The RR time T_{RR} is estimated by evaluating ΔJ_ω , the coherent change in the

² For the parameters of interest here, Lense-Thirring precession is much less efficient than mass and GR precession even for a maximally spinning MBH, and we do not consider it here.

magnitude of the specific angular momentum up to a time t_ω . The change ΔJ_ω is then the step size for the non-coherent growth of the angular momentum over times $t > t_\omega$. Two nearby stars with semi-major axes a exert a mutual specific torque $\sim GM_\star/a$. To zeroth order the torques of the stars on a test wire cancel, so that within a distance a from the MBH the net torque on a test star is determined by the Poissonian excess torque $\dot{J} \sim \sqrt{N(<a)}GM_\star/a$ and

$$\Delta J_\omega \sim \dot{J}t_\omega = \sqrt{N(<a)}(GM_\star/a)t_\omega. \quad (2)$$

For $t > t_\omega$ the torques on a particular star-wire become random, and the change in angular momentum grows in a random walk fashion with a timescale $T_{\text{RR}} \sim (J_c/\Delta J_\omega)^2 t_\omega$, defined as

$$T_{\text{RR}} \equiv A_{\text{RR}} \frac{N(>\mathcal{E})}{\mu^2(>\mathcal{E})} \frac{P^2(\mathcal{E})}{t_\omega} \simeq \frac{A_{\text{RR}}}{N(<a)} \left(\frac{M_\bullet}{M_\star} \right)^2 \frac{P^2(a)}{t_\omega}, \quad (3)$$

where $\mu \equiv NM_\star/(M_\bullet + NM_\star)$, A_{RR} is a numerical factor of order unity, to be determined by simulations, and the last approximate equality holds in the Keplerian regime.

Over most of the relevant phase space the precession is due to the deviations from pure Keplerian motion caused by the potential of the extended stellar cluster. This occurs on a timescale $t_\omega = t_M = [M_\bullet/N(<a)M_\star]P(a)$, assuming $N(<a)M_\star \ll M_\bullet$. The J -averaged RR timescale can then be written as

$$T_{\text{RR}}^M = A_{\text{RR}} \frac{M_\bullet}{M_\star} P(a) = \frac{A_{\text{RR}}}{A_\Lambda} \frac{M_\star}{M_\bullet} N(<a) T_{\text{NR}}. \quad (4)$$

Since $T_{\text{RR}}^M \ll T_{\text{NR}}$ for small a where $N(<a)M_\star \ll M_\bullet$, the RR rate of angular momentum relaxation is much higher than the rate of energy relaxation in the resonant regime. This qualitative analysis has been verified by detailed numerical N -body simulations by Rauch & Tremaine [45] and by Rauch & Ingalls [46].

For most of parameter space, orbital precession is dominated by the mass of the stellar cluster and the RR timescale is well approximated by $T_{\text{RR}} \sim T_{\text{RR}}^M$. However, very close to the MBH, or on wide orbits with very low angular momentum, so that the periaapse is close to the Schwarzschild radius of the MBH, precession is dominated by GR effects. In this case the timescale for precession is given by $t_\omega = t_{\text{GR}} = (8/3)(J/J_{\text{LSO}})^2 P$; here $J_{\text{LSO}} \equiv (4GM_\bullet/c)$ is the angular momentum of the last stable orbit (LSO). When $t_{\text{GR}} \ll t_M$ and GR precession dominates, the RR timescale is (Eq. 4)

$$T_{\text{RR}}^{\text{GR}} = \frac{3}{8} A_{\text{RR}} \left(\frac{M_\bullet}{M_\star} \right)^2 \left(\frac{J_{\text{LSO}}}{J} \right)^2 \frac{P(a)}{N(<a)}. \quad (5)$$

Generally, GR precession and mass precession occur simultaneously, and the scalar RR timescale $T_{\text{RR}}^s(\mathcal{E}, J)$ is given by substituting $1/t_\omega = |1/t_M - 1/t_{\text{GR}}|$ in Eq. (3), where the opposite signs reflect the fact that mass precession is retrograde whereas GR precession is prograde. Thus, the scalar RR timescale is

$$T_{\text{RR}}^s = \frac{A_{\text{RR}}}{N(<a)} \left(\frac{M_\bullet}{M_\star} \right)^2 P^2(a) \left| \frac{1}{t_M} - \frac{1}{t_{\text{GR}}} \right|. \quad (6)$$

We use the relation $d(J^2)/J_c^2 = dt/T_{\text{RR}}^s(\mathcal{E}, J)$ (Eqs. 6) to define the J -averaged time it takes a star to random-walk from $J = J_c(\mathcal{E})$ to the loss-cone $J = J_{lc}$ as

$$\bar{T}_{\text{RR}}^s(\mathcal{E}) = \frac{1}{J_c^2} \int_{J_{lc}^2}^{J_c^2} dJ^2 T_{\text{RR}}^s(\mathcal{E}, J). \quad (7)$$

1.2. Vector resonant relaxation

For time scales much larger than the dynamical time, orbits precess and describe a rosette shape. One can then consider the torques between different rosettes rather than between different wires. Since the rosettes describe planar rings to good approximation, they cannot modify the magnitude of the angular momentum of the star, but they can change the direction of the angular momentum vector. This process is known as “vector resonant relaxation” (Rauch & Tremaine [45]). Vector RR grows coherently ($\propto t$) on timescales $t \ll t_\varphi$, where t_φ is the timescale for a change of order unity in the total gravitational potential φ caused by the changes in the stellar potential φ_\star due to the realignment of the stars as they rotate by π on their orbit,

$$t_\varphi = \frac{\varphi}{\dot{\varphi}_\star} \simeq \frac{N^{1/2} P}{\mu} \frac{P}{2} \simeq \frac{1}{2} \frac{M_\bullet}{M_\star} \frac{P}{N^{1/2}}, \quad (8)$$

the last approximate equality holds for $NM_\star \ll M_\bullet$. In analogy to scalar RR (Eq. 2), the maximal coherent change in \mathbf{J} is $|\Delta \mathbf{J}_\varphi| \sim \dot{J} t_\varphi \sim J_c$, that is, \mathbf{J} rotates by an angle $\mathcal{O}(1)$ already at the coherent phase. On timescales $t \gg t_\varphi$, $|\Delta \mathbf{J}_\varphi|$ cannot grow larger, as it already reached its maximal possible value, but the orbital inclination angle is continuously randomized non-coherently ($\propto t^{1/2}$) on the vector RR timescale (Eq. 3),

$$T_{\text{RR}}^v = 2A_{\text{RR}}^v \frac{N^{1/2}(> \mathcal{E})}{\mu(\mathcal{E})} P(\mathcal{E}) \simeq 2A_{\text{RR}}^v \left(\frac{M_\bullet}{M_\star} \right) \frac{P(a)}{N^{1/2}(< a)}, \quad (9)$$

where the last approximate equality holds for $NM_\star \ll M_\bullet$.

It is that while the torques driving scalar and vector resonant relaxation are the same, vector RR is much more efficient than scalar RR, $T_{\text{RR}}^v \ll \bar{T}_{\text{RR}}^s$, due to the much longer coherence time $t_\varphi \sim N^{1/2} t_M \gg t_M$. Furthermore, vector RR proceeds irrespective of any precession mechanisms that limit the efficiency of scalar resonant relaxation.

2. The origin of the young stellar population in the Galactic center

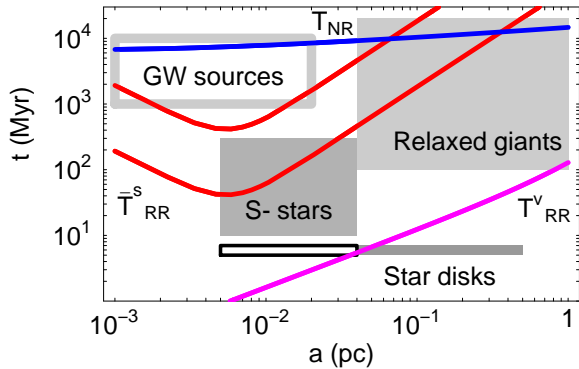


Figure 1. Stellar components, timescales and distance scales in the GC. The NR timescale T_{NR} (top straight line); the timescale \bar{T}_{RR}^s , estimated for $1 M_\odot$ stars (top curved line) and $10 M_\odot$ stars (bottom curved line); the timescale T_{RR}^v (bottom straight line); the position and estimated age of the young stellar rings in the GC (filled rectangle in the bottom right); the position and age of the S-stars if they were born with the disks (empty rectangle in the bottom left); the position and maximal lifespan of the S-stars (filled rectangle in the middle left). Reprinted with permission from the Astrophysical Journal

Figure (1) compares the distance scales and the ages or lifespans of the various dynamical structures and components in the inner pc of the GC with the relaxation timescales. The NR timescale in the GC, which is roughly independent of radius, is $T_{\text{NR}} \sim \text{few} \times 10^9 \text{ yr}$ (Eq. 1). The scalar RR time \bar{T}_{RR}^s is shown for $M_\star = 1, 10 M_\odot$. At large radii the RR time decreases towards the center, but for small radii, where GR precession becomes significant, it increases again. The

vector RR timescale T_{RR}^v , in contrast, decreases unquenched with decreasing radius. Structures with estimated ages exceeding these relaxation timescales must be relaxed.

Two distinct young stellar populations exist in the GC. At distances of 0.04–0.5 pc from the MBH there are about ~ 70 young massive OB stars ($M_\star \gg 10 M_\odot$, lifespan of $t_\star = 6 \pm 2$ Myr), which are distributed in two nearly perpendicular, tangentially rotating disks (Levin & Belobodorov [32]; Genzel et al. [20]; Paumard et al. [43]). It appears that these stars were formed by the fragmentation of gas disks (Levin & Belobodorov [32]; Levin [31]; Nayakshin & Cuadra [40]; Nayakshin & Sunyaev [41]; Nayakshin [42]). Inside the inner 0.04 pc the population changes. There is no evidence for old stars, and the young stars there (the “S-stars”) are main-sequence B-stars ($M_\star \lesssim 15 M_\odot$, lifespans of $10^7 \lesssim t_\star \lesssim 2 \times 10^8$ yr; Ghez et al. [21]; Eisenhauer et al. [13]) on randomly oriented orbits with a random (thermal) J -distribution. There is to date no satisfactory explanation for the presence of the S-stars so close to the MBH (see Alexander [4] for a review).

The existence of coherent dynamical structures in the GC constrains the relaxation processes on these distance scales, since the relaxation timescales must be longer than the structure age t_\star to avoid randomizing it. Figure (1) shows that the observed systematic trends in the spatial distribution, age and state of relaxation of the different stellar components of the GC are consistent with, and perhaps even caused by RR. The star disks are young enough to retain their structure up to their inner edge at 0.04 pc, where $t_\star \sim T_{\text{RR}}^v$ and vector RR can randomize the disk (Hopman & Alexander [27]). It is tempting to explain the S-stars as originally being the inner part of the same disks that are currently present in the GC. However, this scenario is somewhat problematic. First, we note that vector relaxation can only change the inclinations of the orbits, and not their eccentricities, while many of the S-stars have high ($e > 0.9$) eccentricities; the scalar resonant relaxation time is larger than the age of the disks. Second, resonant relaxation alone cannot explain why the S-stars are systematically less massive than the disk stars. An alternative (Levin [33]) would be that the S-stars were perhaps formed in previous accretion disks of which the dynamical signatures have now disappeared.

If the S-stars were *not* formed in the disk, but captured by either a tidal binary disruption (Gould & Quillen [22]; see also contribution from Perets et al. in this volume) or an exchange interaction with a stellar mass black hole (Alexander & Livio [3]), they may be much older than the disks, and in particular their age may be comparable to the local scalar RR time (see figure 1). In this case, RR will redistribute their orbits within their life-time. This may be an essential element of these formation mechanisms: both scenarios lead to rather eccentric orbits (especially tidal binary disruption), whereas not all the orbits of the S-stars are very eccentric: star S1 has eccentricity $e = 0.358 \pm 0.036$, and S31 has $e = 0.395 \pm 0.032$ (Eisenhauer et al. [13]). Since the age of these stars may well exceed the RR time, RR may have redistributed the eccentricities to the current DF, which is consistent with a thermal DF.

Regardless of the origin of the S-stars, their random orbits are consistent with the effect of RR. Vector RR can also explain why the evolved red giants beyond 0.04 pc, in particular the more massive ones with $t_\star \ll \min(T_{\text{NR}}, \bar{T}_{\text{RR}}^s)$ are relaxed, since $T_{\text{RR}}^v < t_\star$ out to ~ 1 pc.

3. Gravitational wave sources

MBHs with masses $M_\bullet \lesssim 5 \times 10^6 M_\odot$ have Schwarzschild radii $r_S = 2GM_\bullet/c^2$, such that a test mass orbiting at a few r_S emits gravitational waves (GWs) with frequencies $10^{-4} \text{ Hz} \lesssim \nu \lesssim 1 \text{ Hz}$, detectable by the planned space based *Laser Interferometer Space Antenna*³ (*LISA*). Such GW sources, for which the mass of the inspiraling object is many orders of magnitude smaller than the mass of the MBH are known as *extreme mass ratio inspiral sources* (EMRIs). GW inspiral events are very rare (of the order of $10^{-7} - 10^{-8} \text{ yr}^{-1}$ per galactic nucleus; e.g. Hils & Bender [23];

³ <http://lisa.jpl.nasa.gov/>

Sigurdsson & Rees [50]; Ivanov [29]; Freitag [17]; Alexander & Hopman [2]; Hopman & Alexander [26], [27], [28]), and it is unlikely that we will observe GWs from our own Galactic center (GC), although such a possibility is not entirely excluded (Freitag [18]; Rubbo, Holley-Bockelmann & Finn [48]). The galactic MBH plays nevertheless a role of importance in understanding the dynamics of EMRIs, since its mass is very close to the mass of the “optimal” *LISA* EMRI target, and as a consequence one may use the GC to model extra-galactic nuclei.

Hopman & Alexander [26] used a model based on the GC to analyze the dynamics of EMRIs. One of the main results was that inspiraling stars always originate very near the MBH, within a distance of ~ 0.01 pc: due to the relatively short relaxation time in galactic nuclei, stars that start to spiral in from larger distances are very likely to plunge into the MBH before becoming observable as GW emitters. This result was confirmed qualitatively by *N*-body simulations by Baumgardt et al. [9] of tidal capture of MS stars by an intermediate mass black hole (Hopman, Portegies Zwart & Alexander [24]; Hopman & Portegies Zwart [25]).

The fact that only stars within ~ 0.01 pc spiral in successfully, implies that it is the stellar content and dynamics of that region which determine the rate of GW inspiral events for the different populations in the system. This means, for example, that mass-segregation is likely to play an important role (Hopman & Alexander [28]; Freitag et al.; [19]; see also contribution by Marc Freitag in this volume). Since the resonant relaxation time is very short ($T_{\text{RR}} \ll T_{\text{NR}}$) near ~ 0.01 pc, it also implies that RR will dictate the rate at which stars are driven towards low *J* orbits, where energy dissipation is efficient and stars spiral in.

Hopman & Alexander [27] used a Fokker-Planck method in energy space with a sink term due to RR losses in *J*-space to calculate the GW inspiral rate. At every time-step, stars redistribute in energy-space due to (non-resonant) two body scattering, and stars are accreted by the MBH with some specified rate per energy bin. In the relevant regime, this rate is assumed to be of order $\sim N(E)/T_{\text{RR}}$, i.e., within one RR time all stars in the bin would be accreted if they were not replaced by new stars that flow to higher energies (tighter orbits). In spite of the fact that stars are drained very efficiently near the MBH, Hopman & Alexander [27] found that the rate at which stars are replenished by two-body scattering is sufficiently high that the stellar distribution will not be depleted near the MBH, unless the efficiency of RR is more than an order of magnitude larger than exploratory *N*-body simulations (Rauch & Tremaine [45]) have indicated. Modifications of the stellar DF due to RR are too small to be observable. Some of the stars that are captured by the MBH are swallowed directly without giving a GW signal, but the stars closed to the MBH (within ~ 0.01 pc) will spiral in rapidly enough to give obtain an orbit of period $P \lesssim 10^4$ s for more than a year. Such sources would be observable to *LISA* to distances up to a few Gpc, depending on the mass of the inspiraling star. Since the enhanced rate at which stars flow to the loss-cone in angular momentum space is sustained by the larger flow due to two-body scattering in energy space, the rate at which EMRIs are produced is increased. The analysis by Hopman & Alexander [27] indicates that the rate at which *observable* EMRIs are formed in galactic nuclei is ~ 8 times higher than that for the case in which RR was neglected.

4. Conclusions

Resonant relaxation is a relatively unexplored dynamical mechanism. Vector RR, which only affects the orientation of the orbit but not the eccentricity, operates in many stellar systems, while scalar RR, which does affect the eccentricity, is unique for MBH systems. This mechanism becomes important at distances $\lesssim 0.1$ pc from the MBH. It may have played an important role in redistributing the orbits of the S-stars, and enhances estimates of the GW inspiral rate by nearly an order of magnitude. Our own GC provides a unique case study for resonant relaxation.

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